1. A large insurance agency services a number of customers who have purchased both a homeowner’s policy and an automobile policy from the agency. For each type of policy, a deductible amount must be specified.

For an automobile policy, the choices are $100 and $250, whereas for a homeowner’s policy, the choices are 0, $100, and $200.

Suppose an individual with both types of policy is selected at random from the agency’s files. Let X = the deductible amount on the auto policy and Y = the deductible amount on the homeowner’s policy.

Possible (X, Y) pairs are then (100, 0), (100, 100),  
(100, 200), (250, 0), (250, 100), and (250, 200); the joint pmf specifies the probability associated with each one of these pairs, with any other pair having probability zero.   
  
Suppose the joint pmf is given in the accompanying joint probability table:

Table

Description automatically generated

1. What is the marginal distribution of X and Y?
2. What is the conditional distribution of X given that Y = 100?
3. Are X and Y independent?
4. A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let X = the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and Y = the proportion of time that the walk-up window is in use.

Then the set of possible values for (X, Y) is the rectangle

D = {(x, y): 0 ≤ x ≤ 1, 0 ≤ y ≤ 1}.

Suppose the joint pdf of (X, Y) is given by

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1. Verify that it is a legitimate pdf
2. What is the probability that neither facility is busy more than one-quarter (0.25) of the time in a day?
3. What is the conditional probability that Y ≤0.5 given that X = .8?
4. What is the marginal probability that Y ≤ 0.5?
5. Are X and Y independent?